

# Significance and Feasibility of a Joint Assessment of Arrival Angles and Times in Experimental Tomography Methodology

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## RESUME

Cette étude a pour but de fournir une synthèse des effets du milieu sur l'efficacité des techniques de traitement d'antenne en milieu perturbé et tout particulièrement dans le cas de l'Atlantique Nord-Est.

Une synthèse des connaissances actuelles doublée d'une approche paramétrique vise à améliorer la compréhension des mécanismes physiques mis en jeu et à spécifier la validité des théories utilisées pour traiter le problème de la propagation des signaux tomographiques en milieu marin aléatoire. Elle permet d'évaluer théoriquement les ordres de grandeur des perturbations causées sur les paramètres inversibles de la tomographie.

Une seconde approche statistique basée sur des simulations numériques en Atlantique Nord-Est permet de vérifier les ordres de grandeur précédents, d'augmenter et d'affiner les informations concernant l'étalement angulaire.

La finalité de ces deux parties est de fournir un diagramme conjoint temps-site en Atlantique Nord-Est. Ces résultats servent de base à la discussion finale de cette étude visant à conclure sur l'intérêt d'une antenne pour estimation conjointe temps-site en tomographie.

## I. INTRODUCTION

Although numerical inversion techniques are becoming more and more performant, their results depend on the accuracy of the measured input data. These data, known as the invertible parameters of tomography (travel time fluctuations of an acoustic pulse, arrival angle...), are deduced from observables identified as a line integral of the desired field.

Fluctuations of the medium noticeably modify the invertible parameters. In a modelization of

## ABSTRACT

The purpose of this study is to theoretically evaluate the performances of a vertical array in a fluctuating medium by the mean of arrival times-angles diagrams.

First, pertinence of using the eikonal approximation to monitor the sound speed structure in a fluctuating ocean and especially to probe meso-structure in the North-East Atlantic Ocean is discussed.

Then, a study of the evolution of tomographic parameters: travel time, temporal spreading, temporal resolution, angular density... based on rays theory is conducted through a synthesis of recent theoretical studies and a parametric approach. Its aim is to quantify and precisely define the undesirable effects induced by the fluctuating medium.

These results are compared with those of a statistical study, and then efficiency of an array processing dealing with a joint assessment of arrival angles and times is investigated.

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acoustic propagation with geometric rays, this generates general problems of identification and separability of acoustic rays. Mathematical inversion methods may be ill-conditioned and it becomes difficult to yield information on ocean structure. This is the case of measurements obtained during the acoustic tomography experiment carried out in 1990 by the SHOM (Hydrographic and Oceanographic Center of French Navy) in the Bay of Biscay. Because of very specific Sound Velocity Profile, received rays are no longer separable and identifiable, and the stability of geometric rays is questionable.

In order to improve tomographic techniques in the North-East Atlantic ocean, the mechanisms of the influence of propagation conditions on tomography results must be well known, summed up and compiled. Furthermore, it seems that non resolvable paths might be separated with a vertical reception array. The purpose of this study is to theoretically evaluate the performances of such an array in a fluctuating medium by the mean of arrival times-angles diagrams.

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Finally some general features of the feasibility of this improvement are pointed out from these first results and some particular conclusions are given in the case of the North-East Atlantic ocean (NEA).

## II. DETERMINISTIC ASPECT

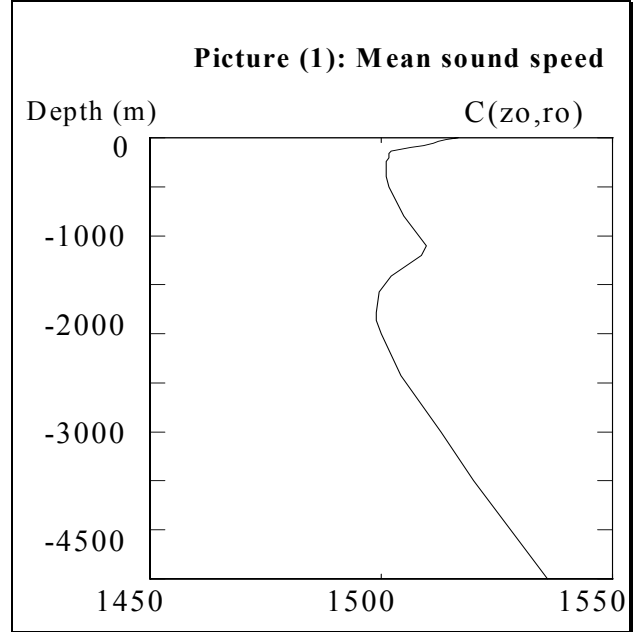
In a first way, we have checked up the validity of rays equations in a double-channel specific of NEA (North-East Atlantic) by considering order of magnitudes of the spatial celerity gradients. They are in fact insufficient to explain instable motion of rays paths.

On the other hand, it has been previously shown that acoustic ray trajectories are expected to exhibit chaotic motion, i.e. extreme sensitivity to initial and environmental conditions, in range-dependent ocean models. J.Yan (1993) defines as follow an instability criterion of Hamilton's ray equations:

$$\frac{C_0^2}{C(z_0)^4} \left\{ C(z_0) \left[ \frac{\partial^2 C(z_0)}{\partial z^2} \right] - 3 \left[ \frac{\partial C(z_0)}{\partial z} \right]^2 \right\} + \frac{\partial^2 g(z_0, r_0)}{\partial z^2} < 0$$

where  $C_0$  is the reference sound speed,  $C$  is the sound speed,  $g$  is the range-dependent perturbation

in potential function,  $z$  is the depth,  $z_0$  and  $r_0$  are the reference depth and range. By analyzing instability he shows that this criterion is a necessary condition for rays chaos, and so that chaos may be induced



even if there is no fluctuation ( $g = 0$ ). The picture (2) presents as a function of depth the evolution of this criterion in a case of a deterministic sound speed profile characteristic of the NEA zone. Even in this deterministic case there are two areas where necessary conditions for chaos are satisfied ( $z \approx 1200$ m (channel axis) and  $z \approx 70$ m).

## III. STOCHASTIC ASPECT

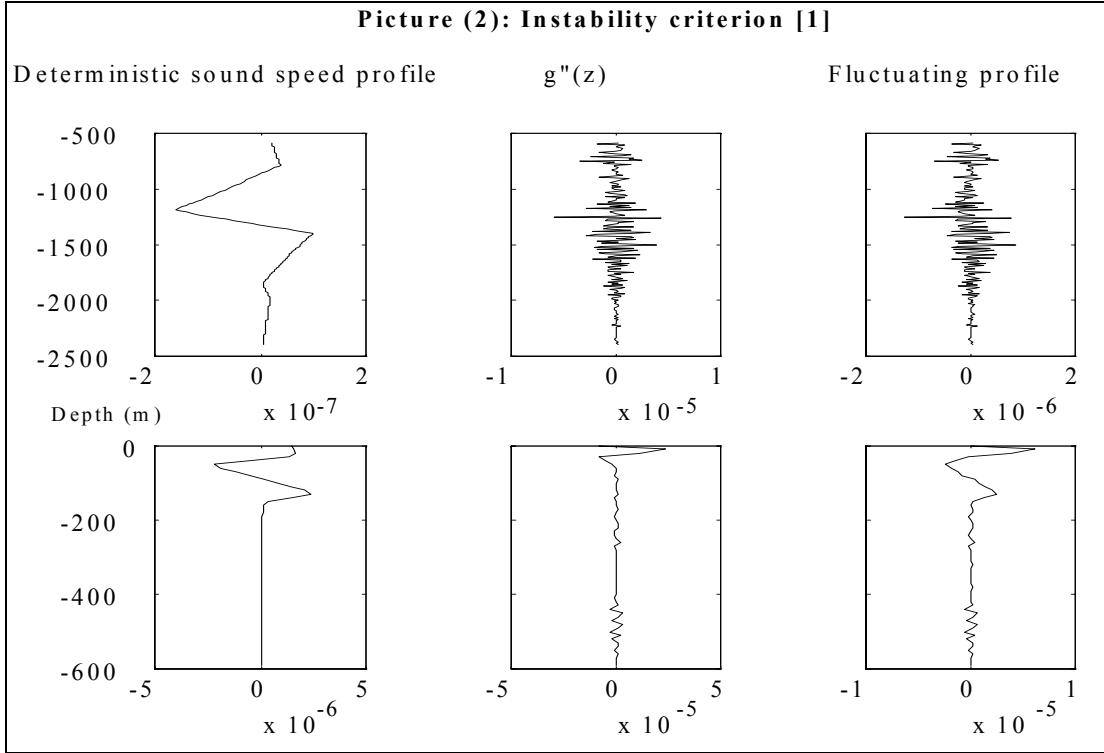
When some fluctuations are introduced in the sound speed profile the criterion is more likely to be satisfied if the derivative  $\partial^2 g / \partial z^2$  is negative and admits a great absolute value. This is the case with NEA fluctuations (see picture (2)) near channel axis and surface of course (due essentially to deterministic aspect) and also now in our interesting depths: near source depth for the tomographic experiment, i.e. between 200m and 400m. This is not a sufficient condition, but the conclusion is that NEA fluctuating profiles authorize the existence of chaotic motions. To investigate and characterize more precisely ray chaos, we should evaluate as in [K. Smith, M. Brown, F. Tappert 1992] the Lyapunov exponents in order to qualify the degree of chaos in this area.

In order to give sound propagation features with rays model in a fluctuating ocean, we use theories developed by Flatté and al and based on path-

integral methods [S. Flatté, R. Dashen, W. Munk, K. Watson, F. Zachariasen (1979)]. Their treatment of ocean medium includes the effects of anisotropy and

the background sound channel as well as statistical inhomogeneity and internal-wave spectra.

**Picture (2): Instability criterion [1]**



They define two parameters representing the strength and size (spatial extent) of the inhomogeneity that control the character of the fluctuations in a wave field crossing these perturbations. These parameters may be evaluated only from in-situ measurements of sound speed fluctuations by the mean of their variance and correlation lengths for some propagation configurations (pair source-receiver).

#### IV. SOUND SPEED FLUCTUATIONS IN NEA

Fluctuations known from in-situ measurements in the NEA zone are compared in their order of magnitude and vertical correlation lengths to those considered in Flatté and al's theories. In fact NEA fluctuations are weaker in the first channel. Because of these orders of magnitude and the closeness of continental shelves, assumption that fluctuations may be essentially due to internal waves seems to be consistent. In addition, we suppose, but with no possible confirmation (because of too sparse in-situ measurements), that the horizontal correlation length was approximately equal to 10 km as it is usually taken for that kind of perturbations.

#### V. CALCULATION OF THE STRENGTH AND DIFFRACTION PARAMETERS $\phi$ AND $\Lambda$ .

The parameters defined as follow:

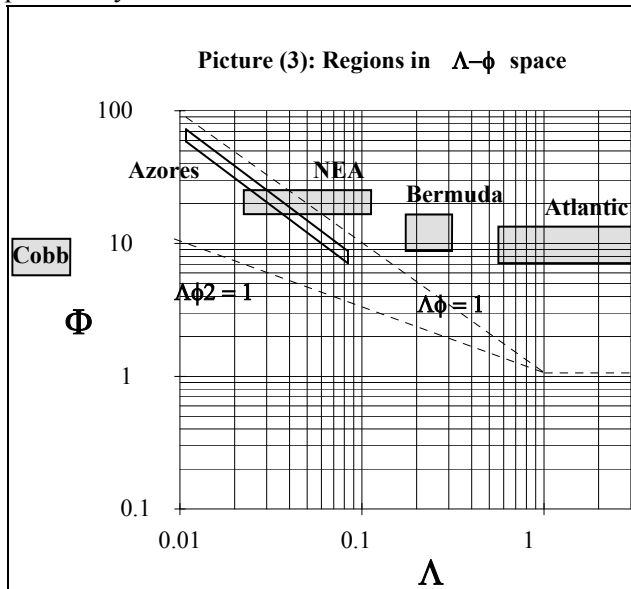
$$\phi^2_{ray} = q_0^2 \int_0^R dx \langle \mu^2(z_{ray}) \rangle \langle L_P(\theta, z_{ray}) \rangle \text{ and}$$

$$\Lambda_{ray} = \phi^{-2}_{ray} q_0^2 \int_0^R dx \langle \mu^2(z_{ray}) \rangle \langle L_P(\theta, z_{ray}) \rangle |q_0 A L_V^2|^{-1}$$

where  $\mu$  is the relative sound speed fluctuation,  $L_p$  the correlation length and  $A$  the phase curvature function, are numerically computed as in [R. Esswein, S. Flatté 1980][R. Leung, H. DeFerrari 1980] along some unperturbed eigenrays localized in the upper channel. We use a Runge-Kutta method to carry on the calculus of the phase curvature function. Some convergence difficulties appear in computing these parameters: their equations are shown to have instability criterion dependent on  $\partial^2 g / \partial z^2$  too.

The results obtained are summarized on picture (3) for a propagation range of 180 km and compared with results in Cobb, Bermuda, Azores in [Flatté et Al] p238 and with Atlantic in [Esswein-Flatté]

p1530. They seem to be in good agreement and coherent with those obtained in the nearest previously studied area: Azores.



So, the whole NEA zone seems to be at its limits of tractability with the basic equation of geometrical optics.

### VI. TIME-ANGLE PARAMETRIC DIAGRAM

These parameters give some information about the number of multipaths and their spatial spreading. They are also used to evaluate temporal and angular spread and wander. Temporal characteristics are presented on picture (4) for a sample of eigenrays. The parametric values obtained are ranged from 10ms to 50ms and correspond to those observed on in-situ measured temporal scheme.

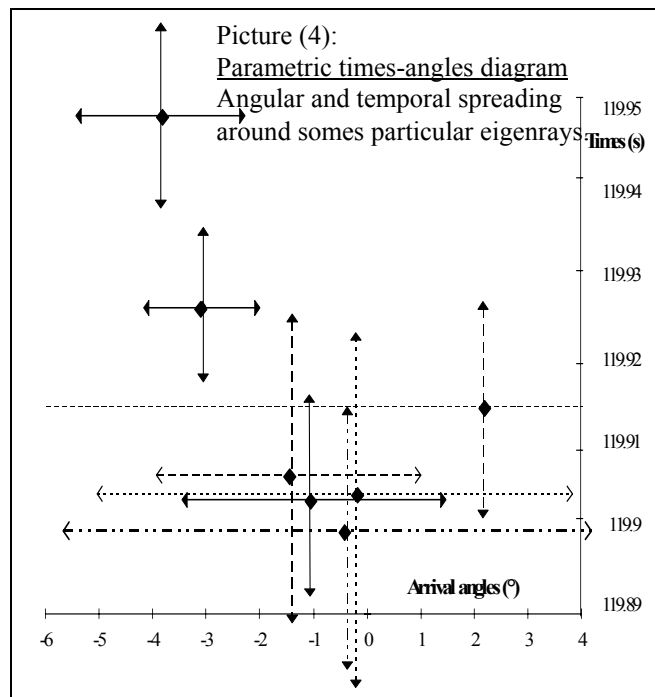
Fluctuations on arrival angle (picture (5)) seem to be very important on the area, especially for axial rays but respect the coherence with the order of magnitude given in [Stoughton-Flatté 1988].

For all of these calculus we take into account the variance of celerity fluctuations over all the NEA area, which is probably the worst case to occur. In order to compare to some measured travel times and arrival angles over 180km range propagation, we have in fact to considered only the variance of the fluctuations of the crossed area instead of the one of the whole zone.

### VII. NUMERICAL SIMULATIONS

This study has been completed with a statistical approach. This consists in constructing from in-situ

celerity measurements, a great number of realistic realizations of the random oceanic process, which are



in fact maps in 2D of the sound speed profile. For each of these maps, the corresponding trajectories and arrival times of rays are then computed by solving a deterministic propagation equation with a range dependent rays model.

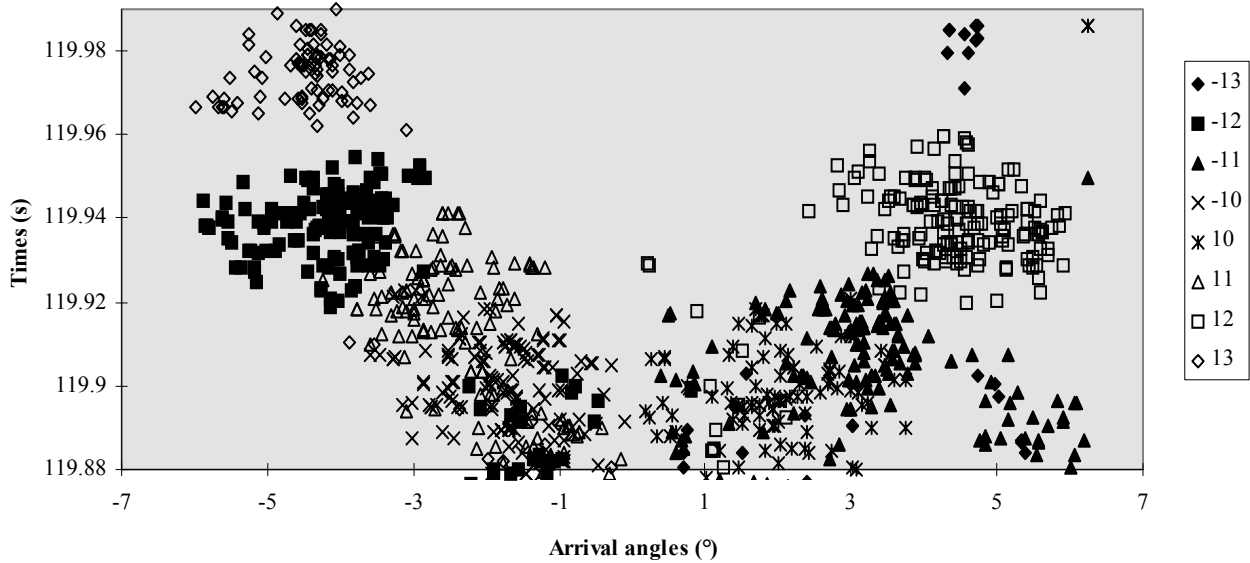
This method makes no assumption on the fluctuations because it only treats deterministic problems and will permit to obtain more realistic results in specific subareas concerning some tomographic paths instead of giving general results for the whole NEA as those of the parametric study.

Sound speed profiles issued from in-situ measurements realized during the GASTOM 90 experiment are sampled each 10 meters in depth. 50 2D descriptions of the sound speed field have been made 10 times by a random choice of some of them among 10 ensembles of decorrelated profiles specific to a subarea. They have been split each 10 km in range. Linear interpolation of gradients has been made between them. These choice satisfy spectral characteristics and continuity of internal waves phenomenon. The program used solves the ray equation numerically with a Runge-Kutta at the 4th order. Convergence of the scheme has been tested numerically, and step have been chosen very often under 10 m because of the instability of this equation due to a fine definition of vertical sound speed

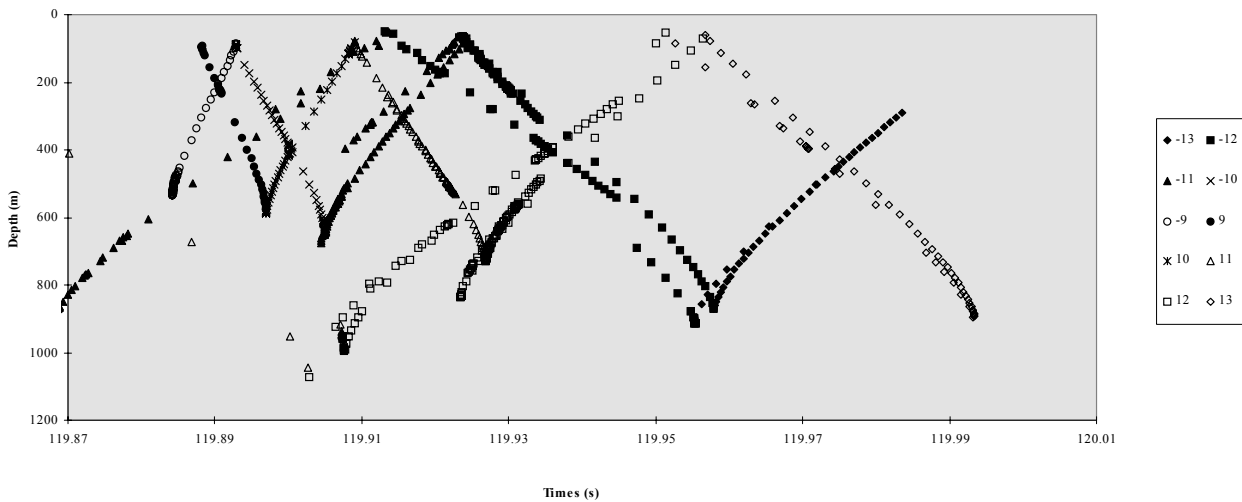
profiles. Vertical localization precision of eigen ray from receiver is less than 0.1 m.

Picture (5) shows an example of a times-angles diagram obtained from 50 simulations.

Picture (5): Times-Angles diagram - Statistical results



Picture (6): One realization of wave fronts simulation - Angular step = 0.01°



Eigenrays obtained from a source at 400 m and a receiver at 400 m depth are referenced by their number of turning points. (For example, ray number -13 admits 13 turning points and has been emitted with a negative angle). Arrival times and angles are different from one realization to the other, and this diagram points out the temporal and angular repartition around a mean value that occurs in a stratified medium. These numerical results have been compared with those of the parametric approach. Temporal spread are in good agreement and have the same order of magnitude as a number of experimental observations.

Statistical results concerning angular repartition seem to be more realistic than the parametric one for small

angles, and are on the same level for higher angles values.

It appears on this picture that the knowledge of arrival time is insufficient to separate and identify the eigenrays. With the knowledge of arrival angle sign, it could be possible, with a temporal resolution of 20 ms to identify rays 13 and -13, and to observed the downward front of the (+/- 12) 's. Table (1) sums up these results and presents for different angular resolutions and receiver depths the rays or fronts that are separable. An angular resolution of 3° seems

theoretically to be optimal and receiver are useful between 400 m and 600 m.

Picture (6) represents the corresponding wave fronts in the equivalent stratified medium, showing the complexity of propagation in such a double channel profile.

Such diagrams have been established for different realizations and highlight distortion (compression and dilation) and rotation of the accordion due to medium fluctuations and mentioned in [Duda 1992]. With a receiver at 400 m depth, the knowledge of arrival angle sign is necessary to separate the conjugated rays  $n$  and  $-n$ .

|       | <b>Angular resolution</b> |   |  |   |
|-------|---------------------------|---|--|---|
|       | no resolution             | 6°                                      | 3°   | 1°  |
| 400 m | 13 Front ↓                | 13<br>-13 Front ↓ 3°<br>-12 Front ↓ -3° | 13<br>-13 Front ↓ 4°<br>12 Front ↑ 4°<br>-12 Front ↓ -4° | 13<br>-13 Front ↓ 4°<br>12 Front ↑ 4°<br>-12 Front ↓ -4°        |
| 600 m | 13 Front ↓                | 13<br>-13 Front ↓ 3°<br>-12 Front ↓ -3° | 13<br>-13 Front ↓ 4°<br>12 Front ↑ 4°<br>-12 Front ↓ -3° | 13<br>-13 Front ↓ 4°<br>12 Front ↑ 4°<br>-12 Front ↓ -4°        |
| 700 m | 13<br>-13 Front ↓         | 13<br>-13 Front ↓ 3°<br>-12 Front ↓ -3° | 13<br>-13 Front ↓ 4°<br>12 Front ↑ 4°<br>-12 Front ↓ -3° | 13<br>-13 Front ↓ 4°<br>12 Front ↑ 4°<br>-12 Front ↓ -4°<br>-11 |

Table (1): Rays or fronts (and their mean arrival angular value) that are separable for different available angular resolutions and receiver depths corresponding to the case of the picture (5). (↓ denotes a downward front)

## VIII. CONCLUSIONS

The parametric study presented here defines the propagation characteristics in the worst fluctuating case. A study of sound speed fluctuations in the NEA area concludes that the orders of magnitude are of the same order than those usually observed in other oceans where rays are stable and identifiable. So, sound speed perturbations can't alone explain the complexity of temporal arrivals detected in-situ during tomographic experiment. Considering chaotic instability, we have shown that the deterministic double-channel satisfies necessary condition for chaos and this aspect is reinforced in case of fluctuating celerity profile.

In conclusion of the theoretical study, problems due to chaos seem to be the more important because chaotic rays may arrive at any time and anywhere and perturb all the arrival scheme without any logic. The interpretation may become then impossible. An issue might be to use eigenrays that do not cross chaotic areas by making judicious choice of source-receiver immersion or by close up the source in its low emission angles.

More information on arrival angles than those available from parametric and bibliographic approach were needed to conclude on the significance of using an array to separate eigenrays in tomographic experiments.

A statistical approach conducted with in-situ observations permits to obtain for different available angular resolutions the number of separable eigenrays for a temporal resolution of 20ms and few receiver depths and so to evaluate gain of a joint assessment of arrival angles and times in tomography experiment which is directly relied to the number of identifiable ray paths that spatially sample in a different way the considered vertical slice of ocean between source and receiver.

## IX. Acknowledgments

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